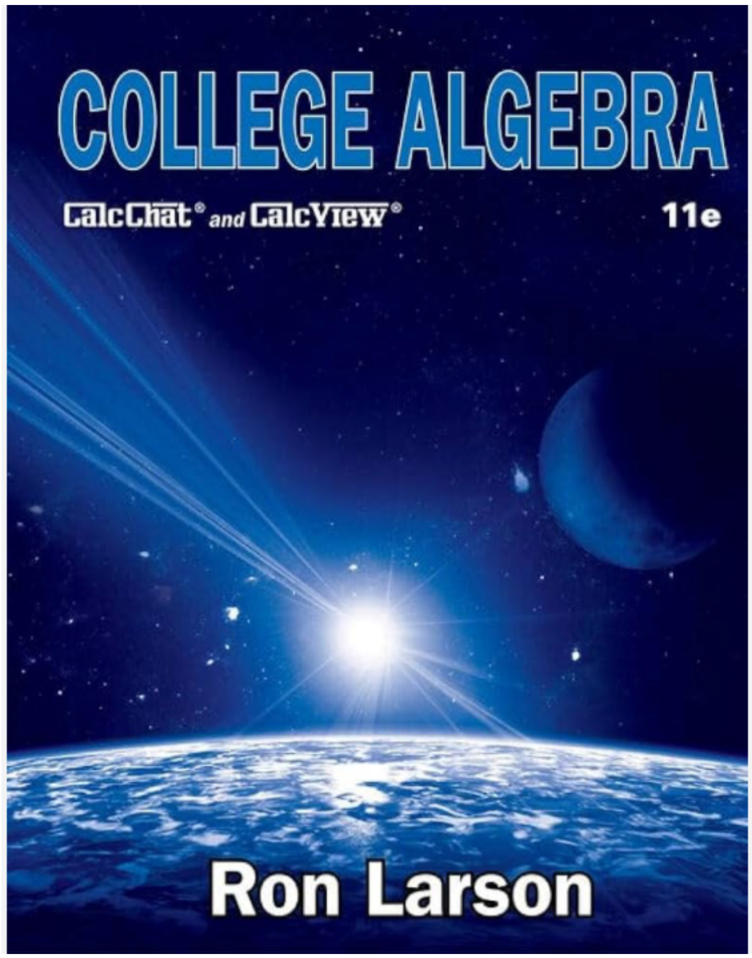
College Algebra



Summer Packet

Review of Geometry, Algebra 1 & 2 Skills

Name: ______________________________

Hello Newest Member of College Algebra!

Welcome to the course! This packet contains all of the skills that you are expected to be able to apply as you enter this challenging, but exciting course. If you lose your packet, please visit the school website to download a new copy.

When completing:

- *Show all of your work within the assignment NEATLY and organized!*
- *To be completed in **PENCIL**.*
- *Show all work for credit.*
- *Questions with NO work will receive NO credit!*
- *This packet is graded based upon correctness.*
- *Box your answers, and **write final answer within packet questions.***
- *NO CALCULATOR UNLESS OTHERWISE STATED!*

This packet is designed with you in mind. Let me take a just a moment of your time to explain!

Each section is titled by a particular skill in **bold** letters. For example, the first section is **Geometry Topics**. After the title, are applicable targets that are the skills you will be applying in the particular section. For example, in the first section you will use the midpoint formula, median of a triangle, perpendicular bisector, altitude of a triangle, equations of lines, and distance formula. After the targets are specific directions/instructions on how to solve problems containing each skill.

These problems (and this packet) are **due on the 3rd day of the school year –** *no exceptions*—to ensure that you have the necessary prerequisite skills in order to be successful in this challenging course. **You will be assessed on this material the first week of school**!

If you have difficulties, you should go online and Google the particular skill you are having trouble with. Well, how do you do that? Use the **bolded** and/or target headings! So, for example, if you are experiencing difficulties on the first section, you should go to Google and type **Midpoint formula.** Another great option is to do the same thing at **Khan Academy** (https://www.khanacademy.org/). There you can get additional explanations through video and practice problems. Khan Academy will be a great resource throughout the year.

Please know that I am looking forward to a great year with you all in college algebra and I will do everything I can to ensure your success. This course will require strong algebra 2 content knowledge and dedication to learning mathematics on a deeper level.

Sincerely,

Ms. Choura

I. Geometry Topics

- **Midpoint formula:** $\left(\frac{x_1+x_x}{2}, \frac{y_1+y_2}{2}\right)$ - **Median of a Triangle:** A segment from a vertex to the midpoint of the opposite side. - **Angle Bisector of a Triangle:** A segment from a vertex that bisects the angle. - **Perpendicular Bisector:** A line passing through the midpoint of and perpendicular to a segment. - **Altitude of a Triangle:** A segment from a vertex perpendicular to the opposite side.	- Equations of Lines: 1. Slope-intercept: $y = mx + b$ where $m = \frac{\Delta y}{\Delta x}$ (Note: here, Δ means "change in." For example, $\Delta y = y_2 - y_1$) 2. Point-slope: $y - y_1 = m(x - x_1)$ 3. Standard form: $Ax + By + C = 0$ - Distance Formula: $d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

Directions – State all linear equations in Slope-Intercept Form unless otherwise stated.

1. Given ΔABC with $A(-5, 4), B(1, 6), C(3, 8)$,write the equation of the median from point C.

2. Write the equation of the line parallel to the line $4x - 6y = -1$ containing the x-intercept of $3x - 2y = 12$

3. Write the equation of the line through $(2, -4)$ and perpendicular to $x - 2y = 7$.

4. Find the value of "a" if a line containing the point $(a, -3a)$ has a y-intercept of 7 and a slope of $-\frac{2}{3}$.

5. Given the distance between $(x, 1)$ and $(-2, 5)$ is $2\sqrt{7}$, find the value(s) of x. Leave your answer in simplified exact form.

6. Write the equation of the perpendicular bisector of the segment joining $A(-5, 4)$ and $B(-3, 6)$.

II. Quadratics/Polynomials

A. Factoring –Strategies to try when factoring:

- Look for a common factor
- Difference of two squares:
  ○ $a^2 - b^2 = (a-b)(a+b)$
- Perfect square trinomial:
  ○ $a^2 \pm 2ab + b^2 = (a \pm b)^2$
	- Factorable trinomial (Target Sum/Target Product)
	- Guess and Check
	- Grouping
	- Sum/Difference of Cubes
	  ○ $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
	  ○ $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

1. *Directions – Factor completely each of the following:*

a. $4x^2 + 27x + 35$

b. $-28y^2 + 7t^2$

c. $x^3 - 2x^2 - 9x + 18$

d. $8a^4 + 27ab^3$

B. Equations – Since the following are equations, we can now go a step further and solve for *x* by factoring or using the quadratic formula.

2. *Directions – Solve each of the following:*

a. $-3x^2 - 5x + 12 = 0$

b. $3x^2 + 5x = 6$

c. $x^2 + 2x + 3 = 0$

d. $225 - b^2 = 0$

C. Graphing – To graph a quadratic equation in standard form, $y = ax^2 + bx + c$, find the important points of the graph by following the steps:

- ***Y*-intercept:** If a point is the y-intercept of the curve, then that is the point at which the graph crosses the y-axis. Since this point is on the y-axis, then the x-coordinate must be 0. Substitute zero in for x and solve for y.
- **Vertex:** x-coordinate of the vertex: $x = -\frac{b}{2a}$.

  y- coordinate of the vertex: substitute the value found for the x-coordinate into the original equation and solve for y: $y = f(x) = f(-\frac{b}{2a})$
- ***x*-intercepts:** if a point is an x-intercept of the curve, then it is a point at which the graph crosses the x-axis. Since these points are on the x-axis, then the y-coordinates must be 0. Substitute zero in for y and solve for x by factoring or using the quadratic formula.

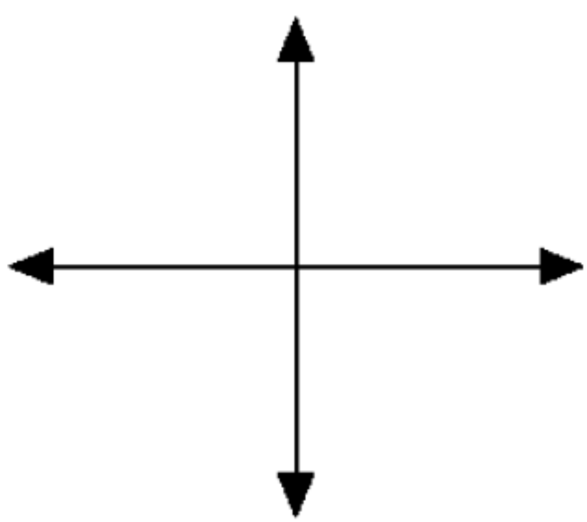
*No calculator, but you should also be able to graph with the use of your calculator.

3. *Directions – Given* $y = -3x^2 - 6x + 2$, *find and graph the following:*

a. y-intercept

b. vertex

c. x-intercepts



III. Systems

Substitution or Linear Combination (Elimination) can be used to solve systems of equations. - -If there is a solution to the system, then the equations are representing intersecting lines. - -If both variables cancel out and an equation is formed that is never true, then there is no solution and the lines never intersect. Lines that never intersect are parallel lines. - -If both variables cancel out and an equation is formed that is always true, then there are infinitely many solutions and the equations must represent the same line. (Coinciding lines)

Directions – Solve each of the following.

- *Explain what the solution tells us about the lines represented by the equations.*
- *No calculator, but you need to be able to solve with the use of a calculator as well.*

1. $\begin{cases} 3x - 4y = 2 \\ -x + 3y = 1 \end{cases}$ Solution: Explanation:	2. $\begin{cases} -x + y = 3 \\ 2x - 2y = -6 \end{cases}$ Solution: Explanation:

IV. Exponents

Directions – Simplify using only positive exponents and no calculator!

Properties:	$a^m \cdot a^n = a^{m+n}$	$(a^m)^n = a^{mn}$	$a^{\frac{p}{r}} = \sqrt[r]{a^p}$
$a^0 = 1, a \neq 0$	$a^{-n} = \frac{1}{a^n}$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\frac{a^m}{b^m} = a^{m-n}$
	$a^{-\frac{p}{r}} = \frac{1}{\sqrt[r]{a^p}}$	$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m = \frac{b^m}{a^m}$	$\frac{a^n b^{-n}}{c^{-m}} = \frac{a^n c^m}{b^n}$

1. $\left(\frac{81}{64}\right)^{\frac{1}{2}}$

2. $(27^{-2})^{\frac{1}{3}}$

3. $\frac{(3x^2)^{-1}}{6x^{-3}}$

4. a. -2^4
   b. $(-2)^4$

5. $\frac{3^{-5} \cdot 3^{-10}}{3^2}$

6. $(4^{-1} \cdot 2^{-1})^2$
- hint 1: $(a^{-m} + a^{-n})^p \neq a^{-mp} + a^{-np}$
-hint 2: Apply the negative exponent property to each term, then get a common denominator, then add.

7. a. $(13y)^{-1}$
   b. $13y^{-1}$

8. $8^{-1} \cdot 8$

V. Logarithms

$\boxed{\text{Given } \log_b a = x \text{ if and only if } b^x = a, \text{ where } b > 0, \text{ but } b \neq 1 \text{ and } a > 0}$

Directions – Solve for x.

1. $3\log_2 x = 12$

2. $\log_5 125 = x$

3. $3 + 4\log_x 4 = 5$

4. $\frac{3}{2}\log_{27}(x + 5) = 1$

5. $1 + \frac{4}{3}\log_{(x-3)} 4 = \frac{11}{3}$

6. $\log_{\sqrt{5}} 25^{4x-1} = 3$

VI. Rational Expressions

Directions – Simplify to a single fraction:

1. Hint: get a common denominator!

$$\frac{1}{ab} - \frac{2}{b^2}$$

2. Hint: factor and cancel!

$$\frac{x^2 + 6x + 8}{x^2 - 4}$$

3. Hint: get a common denominator in the numerator and multiply by the reciprocal, or multiply by the LCD/LCD.

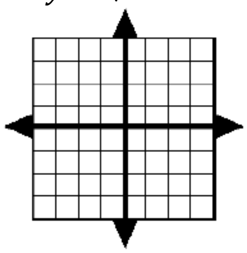
$$\frac{\frac{x}{x-1} + 1}{\frac{x+2}{x}}$$

VII. Quick Graphs:

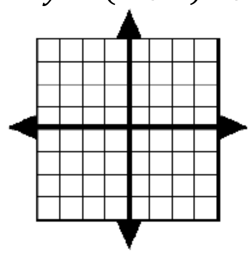
Directions – Graph each of the following.

- *If you don't remember, use your graphing calculator to help you determine the patterns, but you need to be able to do these graphs without your calculator!*

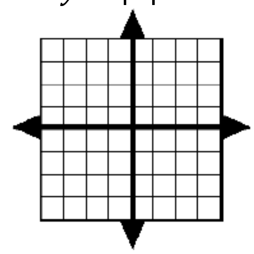
1. $y = \sqrt{x-2} - 3$



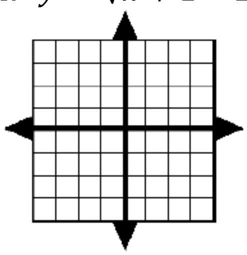
2. $y = (x+2)^2 + 1$



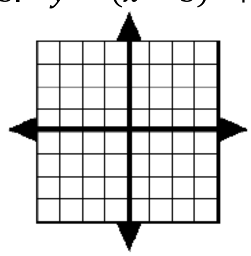
3. $y = |x| - 1$



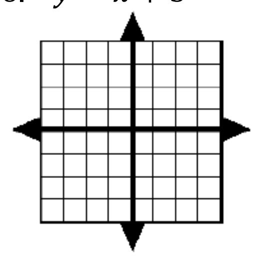
4. $y = \sqrt[3]{x+1} - 2$



5. $y = (x-3)^3 + 2$



6. $y = x + 3$



VIII. Simplifying Radicals

To simplify a radical:

- Find the largest perfect square that will divide evenly into the number under your radical sign.
- If the number under your radical cannot be divided evenly by any of the perfect squares, your radical is already in simplest form and cannot be reduced further.

You should be able to do the following operation in your head!!

Example: $\sqrt{48}$

- Write the number appearing under your radical as the product (multiplication) of the perfect square and your answer from dividing.

  $\sqrt{48} = \sqrt{16 \cdot 3}$

- Give each number in the product its own radical sign.

  $\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3}$

- Reduce the "perfect" radical that you have now created.

  $\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$

- You now have your answer.

  $4\sqrt{3}$

Directions – Simplify each of the radicals without a calculator, include imaginary units when applicable.

1. $\sqrt{27}$
2. $\sqrt{-32}$
3. $\sqrt{\frac{45}{9}}$
4. $\sqrt[3]{128}$
5. $\sqrt[3]{-27}$
6. $3\sqrt{50}$
7. $\sqrt{42}$
8. $\sqrt{18} + \sqrt{8}$

9. $\frac{6}{\sqrt{5}}$

10. $\frac{-3\sqrt{15}}{\sqrt{3}}$

11. $\frac{\sqrt{13}}{4\sqrt{6}}$

12. $7\sqrt{5} - 10\sqrt{5}$

13. $12\sqrt{3} + 3\sqrt{12}$

14. $-6\sqrt{50} + 4\sqrt{32}$

IX. Factoring (continued)

- Look for a common factor
- Difference of two squares:
    ○ $a^2 - b^2 = (a-b)(a+b)$
- Perfect square trinomial:
    ○ $a^2 \pm 2ab + b^2 = (a \pm b)^2$
	- Factorable trinomial (Target Sum/Target Product)
	- Guess and Check
	- Grouping
	- Sum/Difference of Cubes
	    ○ $a^3 + b^3 = (a+b) = (a^2 - ab + b^2)$
	    ○ $a^3 - b^3 = (a-b) = (a^2 + ab + b^2)$

Directions – Factor completely each of the following:

1. $x^2 - 16$

2. $x^2 - 8x + 16$

3. $3x^2 - 5x - 12$

4. $x^2 - 12x + 11$

5. $4x^2 + 2x - 20$

6. $x^2y^2 - 25$

7. $x^2 - 12x + 20$

8. $1 - 9x^2$

9. $7x^2 - 26x - 8$

10. $81x^2 - 4$

11. $x^2 - 9x + 18$

12. $4x^2 - 1$

13. $x^2 + 6x - 40$

14. $c^2 + c - 20$

15. $x^2 + 12x + 35$

16. $16 - 81x^2$

17. $5x^2 - 10x - 15$

18. $6x^2 - 13x + 6$

19. $6x^2 - 15x - 21$

20. $y^2 - 49$

21. $x^2 + 12x + 36$

22. $49x^2 - 36$

23. $3x^2 - 10x + 7$

24. $2x^2 + 9x + 10$

25. $x^2 - x - 6$

26. $2x^2 - 11x - 21$

27. $16x^2 - 121$

28. $x^2 + 9$

29. $x^2y^2 - 100$

30. $y^2 - 13y + 42$

Directions – Solve by factoring.

31. $x^2 - 4x = 0$

32. $y^2 - y - 6 = 0$

33. $3u^2 - 12u + 9 = 0$

34. $6x^2 + 12x = 0$

35. $y^2 + 49y = 0$

36. $y^2 + 5y - 6 = 0$

37. $x + 8 = x(x + 3)$

38. $a^2 - 36 = 0$

39. $a^2 - 7a = -12$

40. $y^2 + 9y = 0$

41. $2x^2 + x = 6$

42. $y^2 - 8y + 12 = 0$

43. $y^2 + 15 = 8y$

44. $x^2 - 6x + 5 = 0$

45. $x^2 + 7x = 0$

46. $x - 6 = x(x - 4)$

47. $5a^2 + 25a = 0$

48. $3x^2 - 9x = 0$

49. $4x^2 + 16x = 0$

50. $x - 25 = x(x - 9)$